

6.4b Exponential Growth and Decay  $\frac{dT}{dt}$ 

Newton's Law of Cooling: The rate at which an object's temperature is changing is directly proportional to the difference between its temperature and the temperature of the surrounding medium.

$T$  = Temp  $T_s$  = Temp of Surrounding  
 $t$  = time

$$\frac{dT}{dt} = -k(T - T_s) \quad e^{\ln|T-T_s|} = e^{-kt+c}$$

$$\int \frac{dT}{T-T_s} = \int -k dt \quad |T-T_s| = e^{-kt} \cdot e^c = e^{-kt} \cdot A$$

$$t=0 \quad T=T_0 \quad \text{beginning Temp}$$

$$T_0 - T_s = e^0 \cdot A = A$$

$$T - T_s = (T_0 - T_s) e^{-kt}$$

$$T = (T_0 - T_s) e^{-kt} + T_s$$

$$v = 105(.997)^x$$

$$y = ab^x$$

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A hard boiled egg at 98 degrees Celsius is put in a pan under running 18 degree water to cool. After 5 minutes, the egg's temperature is found to be 38 degrees. How much longer will it take the egg to reach 20 degrees?

$$T = (T_0 - T_s) e^{-kt} + T_s$$

$$T = (98 - 18) e^{-kt} + 18 \quad 20 = 80e^{-.271t} + 18$$

$$38 = 80e^{-k \cdot 5} + 18 \quad t = 13$$

$$20 = 80e^{-5k}$$

$$13 - 5 = 8 \text{ min longer}$$

$$\frac{20}{80} = e^{-5k}$$

$$\ln \frac{1}{4} = -5k$$

$$k = \frac{\ln .25}{-5} \approx .271$$

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Newton's Law of Cooling

A cup of hot water was placed in the fridge. The temperature was checked at somewhat regular intervals. See the data on the next page. Make a scatter plot and find the exponential regression equation.

time	temp
0	124
5	118
10	114
16	109
20	106

$T_0 = 124 \quad T_s = 45$

$106 = 79e^{-k \cdot 20} + 45$

$T = (124 - 45)e^{-kt} + 45 \quad k = .013$

$T = 79e^{-.013t} + 45 = 79(.987)^t + 45$

calc  $T = 70.8(.990833)^x + 45$

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## Separation of Variables

Solve for y if  $\frac{dy}{dx} = (xy)^2$  and  $y=1$  when  $x=1$

RTC!!

$$\frac{dy}{dx} = x^2 y^2$$

$$-y^{-1} = \frac{x^3}{3} + C$$

$$-1 = \frac{1}{3} + C \quad C = -\frac{4}{3}$$

$$\int \frac{dy}{y^2} = \int x^2 dx$$

$$\frac{-1}{y} = \frac{x^3}{3} - \frac{4}{3} = \frac{x^3 - 4}{3}$$

$$\int y^2 dy = \int x^2 dx$$

$$\frac{1}{y} = \frac{-x^3 + 4}{3}$$

$$y = \frac{3}{4 - x^3}$$

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